Did people in the Old Testament time need to know about Jesus to be saved? If not, why do Christians today say one must have explicit faith in Jesus to be saved?

While salvation has always been by God's grace through the work of Christ on the cross, the human obligation and result associated with this grace has been faith in the revelation about the promise of this grace. As God has revealed himself progressively over time [through biblical history], the content of this faith has grown more explicit with God's revelation.

Adam and Eve had only the promise of an ultimate deliverer [Genesis 3.15], and signaled their faith most explicitly by accepting God's judgment of shedding innocent animal blood to provide their skins, which set a pattern of atoning sacrifices that was carried on [Genesis 4.4]. By the time of Abraham, the revelation had grown to include the promise of deliverance through the line of Abraham [Genesis 12.3; 15.6]. Abraham ultimately showed his faith in this compound promise of a deliverer through his line when he obediently offered Isaac as a sacrifice [Genesis 22.1-18]. So even before the Law of Moses, there was a promise of ultimate deliverance and a sacrificial system of temporary atonement. Through Moses, God brought the Law which further revealed God's promises and codified the sacrificial system. People responded in faith by acting on the promise and taking the sacrifices seriously. During the time of the kings, many prophets revealed details about the coming Deliverer and his deliverance, increasing the revelation of God which required faith.

When Jesus walked the earth, he was the promised Deliverer, the ultimate revelation of God about his promise of deliverance. Jesus was the prophesied descendent of Eve who would smash Satan, the promised greater prophet and greater Moses, the son of David who would reign, the suffering servant, the Messiah. Those who had faith in God's revelation to this point could see clearly the evidences Jesus gave for his identity and could believe his teachings about what he offered in the way of deliverance. Thus, faith in Jesus himself became the obligation and result for man of the grace of God. The only further revelation about God's deliverance was provided by God through the New Testament writers after Jesus' resurrection, and that revelation serves to clarify the identification of Jesus as the promised deliverer, the work of deliverance God offers man through Jesus, and the expectation of God for man's faith in Jesus as the only way to salvation [from man's perspective; keeping in mind it is by grace that man can believe].

Thus, for a person in this dispensation after the New Testament events and writings, faith in Jesus is the obligation and result of God's grace, faith in Jesus is the only manifestation of true faith in God's promise of deliverance. Thus Christians today are correct in asserting that explicit faith in the identity and work of Christ is necessary for salvation.